

Volatility transmission patterns and terrorist attacks

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Volatility transmission patterns and terrorist attacks

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Volatility transmission patterns and terrorist attacks

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Abstract

The objective of this study is to analyze volatility transmission between the US and Eurozone stock markets considering the financial market responses to the September 11, March 11 and July 7 terrorist attacks. In order to do this, we use a multivariate GARCH model and take into account the asymmetric volatility phenomenon, the non-synchronous trading problem and the turmoil periods themselves. Moreover, a graphical analysis of the Asymmetric Volatility Impulse-Response Functions (AVIRF) is introduced, which takes into consideration the financial market responses to the terrorist attacks. Results suggest that there is bidirectional and asymmetric volatility transmission and show the different impact that terrorist attacks had on both markets.

JEL Classification: C32; F30; G15

Keywords: international financial markets; stock market crisis; multivariate GARCH; volatility spillovers

1 Introduction

On September 11, 2001; March 11, 2004; and July 7, 2005, the cities of New York, Madrid and London experienced respectively devastating terrorist attacks. These attacks had an influence over several economic variables and they obviously affected financial markets. Taking into account increasing global financial integration, an important question arises: How did these terrorist attacks affect interrelations between financial markets?

The main objective of this study is to analyze how volatility transmission patterns are affected by stock market crises. Moreover, we compare the different reactions of the markets to the terrorist attacks considered. To do this, we use a multivariate GARCH model and take into account both the asymmetric volatility phenomenon and the non-synchronous trading problem. In our empirical application, we focus on stock market crises as a result of terrorist attacks and analyze international volatility transmission between the US and Eurozone financial markets.

Pericoli and Sbracia (2003)'s survey on financial contagion compiles the five most common definitions for financial crisis contagion.¹ Our work casts with the second definition given by Pericoli and Sbracia (2003, page 574): "Contagion occurs when volatility of asset prices spills over from the crisis country to other countries". The empirical specification used for testing financial crisis contagion can be seen as a special case of the general framework proposed in Dungey et al. (2005), where contagion is represented by a significant response to contemporaneous shocks coming from another market after conditioning on the interdependence between markets in a non-crisis environment.

¹ See also Allen and Gale (2000) for theoretical models on financial contagion.

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It must be highlighted that most existing studies on spillovers between developed countries focus on individual countries such as US, Canada, Japan, UK, France, and Germany.² As far as we know, there are no articles analyzing volatility transmission patterns between the US and the Eurozone as a global market. Moreover, this paper will be the first to take into account the non-synchronous trading problem and to use a sample period that includes the September 11, March 11 and July 7 terrorist attacks.

As far as we know, no paper has analyzed until now the effects of the attacks of March 11 and July 7. Moreover, few studies have examined the effects of the attacks of September 11 on financial markets and they focus on the economy as a whole or in different specific aspects of the economy.³ For instance, Poteshman (2006) analyzes whether there was unusual option market activity prior to the terrorist attacks. Ito and Lee (2005) and Blunk et al. (2006) assess the impact of the September 11 attack on US airline demand. Glaser and Weber (2006) focus on how the terrorist attack influenced expected returns and volatility forecasts of individual investors. Chen and Siems (2004) investigate if terrorist and military attacks (including the September 11 attack) are associated with significant negative abnormal returns in global capital markets. Finally, Choudhry (2005) investigates the effects of the September 11 attack and the period afterwards on the time-varying beta of a few companies in the US. However, none analyze volatility transmission patterns and how they have been affected by the event. As far as we know, the only papers that analyze changes in interrelations between stock markets are Hon et al. (2004) and Mun (2005), but they test whether the terrorist attack resulted in a change in correlation across

² See Koutmous and Booth (1995), Karolyi (1995), Karolyi and Stulz (1996), Darbar and Deb (1997), Ramchand and Susmel (1998), Brooks and Henry (2000), Longin and Solnik (2001), Martens and Poon (2001) and Bera and Kim (2002), *inter alia*.
³ A special issue of the *Economic Policy Review* of the Federal Reserve Bank of New York (2002, Volume 8, Number 2) analyzes general economic consequences of September 11. A special issue of the *Journal of Risk and Uncertainty* (2003, Volume 26, Numbers 2/3) deals with the risks of terrorism with a special focus on September 11. A special issue of the *European Journal of Political Economy* (2004, Volume 20, Issue 2) deals with the economic consequences of terror.

global financial markets. We try to answer the following question: Were there differences in the reaction of the US and Eurozone stock markets to the different terrorist attacks considered? To do so, we propose a new version of the Asymmetric Volatility Impulse Response Functions (AVIRF) which takes into account stock market crises. In particular, we expect to find differences in stock market reactions to the three terrorist attacks. These differences might be due to: i) the impact those attacks had on particular industries/sectors, ii) whether the attack was considered as a local, regional, or global shock, and iii) the state of the economy at the time the event took place.⁴

When studying asset price comovements and contagion between different financial markets, an important fact to take into account are the trading hours in each market. In the case of partially overlapping markets (such as the US and Eurozone), a jump in prices can be observed in the first market to open when the second market starts trading, reflecting information contained in the opening price. Therefore, this could increase volatility in this first market. Moreover, as suggested by Hamao *et al.* (1990), a correlation analysis between partially overlapping markets using close to close returns could produce false spillovers, both in mean and volatility. This is because it is difficult to separate effects coming from the foreign market from those coming from the domestic market while it remains closed.

There are several solutions for artificially synchronizing international markets. First of all, in the case of US, information transmission with other markets can be analyzed through American Depositary Receipts (ADRs), which share trading hours with the US market. The problem is that there are few ADRs, they are not actively traded, and there are microstructure differences between the North American stock market and the original country market [see

⁴ The use of local, regional and global shock is similar to the taxonomy for crisis transmission proposed in Dungey and Martin (2007). These authors propose a model which captures a range of common factors including global shocks, country and market shocks, and idiosyncratic shocks.

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Wongswan (2006)]. Some studies, such as Longin and Solnik (1995) and Ramchand and Susmel (1998), use weekly or monthly data to avoid the non-synchronous trading problem. However, the use of low frequency data leads to small samples, which is inefficient for multivariate modeling. Some studies, such as Hamao et al. (1990) and Koutmos and Booth (1995), use daily non-synchronous open-to-close and close-to-open returns. Nevertheless, these studies cannot distinguish volatility spillovers from contemporaneous correlations. Finally, Martens and Poon (2001) use 16:00-to-16:00 synchronous stock market series to solve this problem. By doing this, they find a bidirectional spillover between US and France and between US and UK, contrary to previous studies that only found volatility spillovers from the US to the other countries. Similarly, Kleimeier et al. (2008) uses synchronous data (they refer to it as time-aligned data) to analyse the financial turmoil surrounding the Asian crisis.

This study innovates with respect the existing literature in two ways. Firstly, we study volatility transmission between the US and the Eurozone using a sample period that includes the terrorist attacks in New York, Madrid, and London. Secondly, we introduce a new version of Asymmetric Volatility Impulse Response Functions which takes into account stock market crises.

The rest of the paper is organized as follows. Section 2 presents the data and offers some preliminary analysis. Section 3 deals with the econometric approach and introduces the AVIRF with crises. Section 4 presents the empirical results and, finally, Section 5 summarizes the main results.

2 Data

The data consists of simultaneous daily stock market prices recorded at 15:00 GMT for the US (S&P500 index) and the Eurozone (EuroStoxx50 index).⁵ At that time, the European markets are about to close and the US market has just started trading. We use stock market prices recorded at 15:00 GMT, at the midpoint of the overlapping hours, to avoid using index prices recorded exactly at the open (US) and close (Eurozone) of trading.

The data is extracted from Visual Chart Group (www.visualchart.com) for the period January 18, 2000 to January 25, 2006. When there is no common trading day, because of a holiday in one of the markets, the index values recorded on the previous day are used.⁶

Table 1. Summary statistics

	$R_{1,t}$	p -value	$R_{2,t}$	p -value
Mean	-0.00009		-0.00019	
Variance	0.00013		0.00021	
Skewness	0.11202	[0.0701]	0.00400	[0.9484]
Kurtosis	3.72923	[0.0000]	4.90041	[0.0000]
Bera-Jarque	782.423	[0.0000]	910.341	[0.0000]
Q(12)	23.2728	[0.0255]	28.8222	[0.0041]
Q ² (12)	502.408	[0.0000]	842.236	[0.0000]
ARCH(12)	185.035	[0.0000]	255.721	[0.0000]
ADF(4)	-1.87522	[0.3443]	-1.52663	[0.5200]
PP(7)	-1.90664	[0.3295]	-1.53550	[0.5154]

Note: p-values displayed as [.]. $R_{1,t}$ and $R_{2,t}$ represent the log-returns of the S&P500 and the EuroStoxx50 indexes. The Bera-Jarque statistic tests for the normal distribution hypothesis and has an asymptotic distribution $X^2(2)$. Q(12) and Q²(12) are Ljung-Box tests for twelfth order serial correlation in the returns and squared returns. ARCH(12) is Engle's test for twelfth order ARCH, distributed as $X^2(12)$. The ADF (number of lags) and PP (truncation lag) refer to the Augmented Dickey and Fuller (1981) and Phillips and Perron (1988) unit root tests. Critical value at 5% significance level of MacKinnon (1991) for the ADF and PP tests (process with intercept but without trend) is -2.86.

⁵ The Dow Jones EURO STOXX 50 Index, Europe's leading blue-chip index for the Eurozone (that is, the European Monetary Union), provides a blue-chip representation of supersector leaders in the Eurozone. The index covers 50 stocks from 12 Eurozone countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain.

⁶ As the New York Stock Exchange (NYSE) was closed during September 11 and the three following days, the S&P500 index value recorded the previous day is used for those dates.

Table 1 presents some summary statistics on the daily returns, which are defined as log differences in index values. The Jarque-Bera test rejects normality of the returns for both indexes. This is caused mainly by excess kurtosis, suggesting that any model for equity returns should accommodate this characteristic of equity returns. The ARCH test reveals that the returns exhibit conditional heteroskedasticity, while the Ljung-Box test (of twelfth order) indicates significant autocorrelation in both markets in squared returns but not in levels. Fat tails and non-normal distributions are common features of financial data. Finally, both the augmented Dickey Fuller (ADF) and Philips and Perron (PP) tests indicate that both series have a single unit root. Table 2 shows that both series (in log levels) are not cointegrated, the optimal lag length following the AIC criterion being four.⁷

Table 2. Johansen (1988) tests for cointegration

Lags	Null	$\lambda_{trace}(r)$	Critical Value	$\lambda_{max}(r)$	Critical Value
4	R = 0	11.81020	20.26184	7.685361	15.89210
	R = 1	4.124843	9.164546	4.124843	9.164546

Note: The lag length is determined using the AIC criterion. $\lambda_{trace}(r)$ tests the null hypothesis that there are at most r cointegration relationships against the alternative that the number of cointegration vectors is greater than r . $\lambda_{max}(r)$ tests the null hypothesis that there are r cointegration relationships against the alternative that the number of cointegration vectors is greater than $r + 1$. Critical values at the 0.05 level are from MacKinnon-Haug-Michelis (1999).

Each terrorist attack had a different effect on financial markets. If we focus on the September 11 attack, both price indexes reached their minimum level on September 21. In the Eurozone, the EuroStoxx50 fell by 6.7% on the day of the attack; and between September 11 and September 21 was down 17.9%. The New York Stock Exchange did not open until September 17

⁷ Cointegration between both indexes was also rejected at the usual significance levels when using closing values of both indexes instead of prices recorded at 15:00 GMT.

and fell by 5.1%. Between that day and September 21, the S&P 500 fell by 12.3%. In contrast with the effects of the September 11, the March 11 terrorist attack affected both markets less. The EuroStoxx50 decreased by 3.1% on the day of the attack and, at the end of that month, it had returned to the pre-attack levels. In the same way, the S&P 500 suffered a small decline (1.5%) and recovered in less than a month. Finally, the July 7 attack had no effect on the S&P 500 and its impact on the EuroStoxx50 was small (1.7%). All in all, the three terrorist attacks affected the Eurozone more than the US market.

Table 3 evaluates the US\$ value impact of the terrorist attack in the analysed markets. It shows that the \$/€ exchange rate was not appreciably affected by the terrorist attacks and, consequently, stock index market returns would not significantly change for US\$ valued portfolios representing stock market indices.⁸

Table 3. Exchange rate and stock index returns in the crisis periods

Crisis Period	Exchange Rate (\$/€)	EuroStoxx 50	S&P 500
Sep 11 – Sep 21, 2001	1.6%	-17.9%	-12.3%
Mar 11 – Mar 22, 2004	-2.6%	-7.4%	-2.5%
July 7, 2005	0.1%	-1.7%	0.2%

Note: This table sums up the returns of the \$/€ exchange rate and the stock indices (Euro Stoxx 50 and S&P 500) for each terrorist attack.

3 The Econometric Approach

3.1 The model

⁸ The authors thank one of the referees for this comment.

The econometric model is estimated in a three-step procedure. First, a VAR model is estimated to clean up any autocorrelation behaviour. Then, the residuals of the model are orthogonalized. These orthogonalized innovations have the useful property that they are uncorrelated across both time and markets. Finally, the orthogonalized innovations will be used as an input to estimate a multivariate asymmetric GARCH model.

To take into account the September 11, March 11 and July 7 terrorist attacks, three dummy series are introduced in the conditional mean equations. These dummies equal one the days following the terrorist attacks in New York, Madrid, and London respectively until the days when the indexes take their lowest values, and 0 otherwise.

Equation (1) models the mean equation as a VAR(5) process:

$$\begin{aligned} R_{1,t} &= \mu_1 + x_1 S11_t + y_1 M11_t + z_1 J7_t + \sum_{p=1}^5 d_{11,p} R_{1,t-p} + \sum_{p=1}^5 d_{12,p} R_{2,t-p} + u_{1,t} \\ R_{2,t} &= \mu_2 + x_2 S11_t + y_2 M11_t + z_2 J7_t + \sum_{p=1}^5 d_{21,p} R_{1,t-p} + \sum_{p=1}^5 d_{22,p} R_{2,t-p} + u_{2,t} \end{aligned} \quad (1)$$

where $R_{1,t}$ and $R_{2,t}$ are US and Eurozone returns, respectively, μ_i, x_i, y_i, z_i and $d_{ij,p}$ for $i,j=1,2$ and $p=1, \dots, 5$ are the parameters to be estimated and $S11_t, M11_t$ and $J7_t$ are dummy series for the terrorist attacks. Finally, $u_{1,t}$ and $u_{2,t}$ are the non-orthogonal innovations. The VAR lag has been chosen following the AIC criterion.

The innovations $u_{1,t}$ and $u_{2,t}$ are non-orthogonal because, in general, the covariance matrix $\Sigma = E(u_t u_t')$ is not diagonal. Following Baele (2005), because the estimated US and Eurozone shocks from the first step could be driven by common news, the non-orthogonal

innovations ($u_{1,t}$ and $u_{2,t}$) are orthogonalized ($\varepsilon_{1,t}$ and $\varepsilon_{2,t}$) in a second step. If we choose any matrix M so that $M^{-1} \sum M'^{-1} = I$, then the new innovations:

$$\varepsilon_t = u_t M^{-1} \quad (2)$$

satisfy $E(\varepsilon_t \varepsilon_t') = I$. Such a matrix M can be any solution of $MM' = \Sigma$.

To model the conditional variance-covariance matrix we use an asymmetric version of the BEKK model [Baba *et al.* (1989), Engle and Kroner (1995) and Kroner and Ng (1998)]. As in the mean equations, we introduce dummy series to take into account the terrorist attacks.

The compacted form of this model is:

$$H_t = C'C + B'H_{t-1}B + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + G'\eta_{t-1}\eta_{t-1}'G + S'\delta_{t-1}\delta_{t-1}'S + M'\xi_{t-1}\xi_{t-1}'M + L'\vartheta_{t-1}\vartheta_{t-1}'L \quad (3)$$

where C , B , A , G , S , M and L are matrices of parameters to be estimated, being C upper-triangular and positive definite, and H_t the conditional variance-covariance matrix in t .

In the bivariate case, the BEKK model is written as follows:

$$\begin{aligned} \begin{bmatrix} h_{11t} & h_{12t} \\ \cdot & h_{22t} \end{bmatrix} &= \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ \cdot & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \\ &+ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \cdot & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} \eta_{1,t-1}^2 & \eta_{1,t-1}\eta_{2,t-1} \\ \cdot & \eta_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} + \\ &+ \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} \delta_{1,t-1}^2 & \delta_{1,t-1}\delta_{2,t-1} \\ \cdot & \delta_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} + \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \xi_{1,t-1}^2 & \xi_{1,t-1}\xi_{2,t-1} \\ \cdot & \xi_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} + \\ &+ \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} \vartheta_{1,t-1}^2 & \vartheta_{1,t-1}\vartheta_{2,t-1} \\ \cdot & \vartheta_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix} \end{aligned} \quad (4)$$

where $c_{i,j}$, $b_{i,j}$, $a_{i,j}$, $g_{i,j}$, $s_{i,j}$, $m_{i,j}$ and $l_{i,j}$ for all $i,j=1,2$ are parameters, $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are the orthogonalized innovation series coming from equation (2), $\eta_{1,t} = \max[0, -\varepsilon_{1,t}]$ and $\eta_{2,t} = \max[0, -\varepsilon_{2,t}]$ are the Glosten *et al.* (1993) dummy series collecting a negative asymmetry from the shocks and, finally, $h_{ij,t}$ for all $i,j=1$, are the conditional second moment series. Similarly to $\eta_{i,t}$, the variables $\delta_{i,t}$, $\xi_{i,t}$ and $\vartheta_{i,t}$ for all $i=1,2$ are the dummy series for the terrorist attacks. They take the values of the shocks in the days following the terrorist attacks in New York, Madrid, and London respectively, until the days where the indexes reach their lowest values and 0 otherwise.

The study of any financial crisis contagion requires first an unambiguous identification of financial crisis. Following Pericoli and Sbracia (2003, page 573), the literature has related a stock market crisis with a sharp fall in the stock market index or with an upsurge in its volatility. Consequently, the most used standard identification pattern is that periods of crisis coincide with periods of extreme values of the variable under consideration (see Pericoli and Sbracia (2003, page 579)). Following this criterion, the presented results correspond to crisis periods defined as follows: from September 11 to September 21 for the New York attack, from March 11 to March 22 for the Madrid attack and July 7 for the London attack. Nevertheless, other possibilities were explored. Results, available upon request, show that coefficients in matrices S, M and L are less significant as the crisis period increases. Alternatively, Caporale *et al.* (2005) proposed a bootstrapping test to select endogenously the breakpoints corresponding to the beginning of the contagion period. The sophisticated method proposed by Caporale *et al.* (2005) is especially suitable for those crises where it is difficult to identify the starting time of crisis contagion but this does not apply to terrorist attacks.

Equation (4) allows for both own-market and cross-market influences in the conditional variance, therefore enabling the analysis of volatility spillovers between both markets. Moreover, the BEKK model guarantees by construction that the variance-covariance matrix will be positive definite.

In equation (4), parameters $c_{i,j}$, $b_{i,j}$, $a_{i,j}$, $g_{i,j}$, $s_{i,j}$, $m_{i,j}$ and $l_{i,j}$ for all $i,j=1,2$ cannot be interpreted individually. Instead, we have to interpret the non-linear functions of the parameters which form the intercept terms and the coefficients of the lagged variances, covariances, and error terms. We follow Kearney and Patton (2000) and calculate the expected value and the standard error of those non-linear functions. The expected value of a non-linear function of random variables is calculated as the function of the expected value of the variables, if the estimated variables are unbiased. To calculate the standard errors of the function, a first-order Taylor approximation is used. This linearizes the function by using the variance-covariance matrix of the parameters as well as the mean and standard error vectors.

The parameters of the bivariate BEKK system are estimated by maximizing the conditional log-likelihood function:

$$L(\theta) = -\frac{TN}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \left(\ln |H_t(\theta)| + \varepsilon_t' H_t^{-1}(\theta) \varepsilon_t \right)$$

where T is the number of observations, N is the number of variables in the system, and θ denotes the vector of all the parameters to be estimated. Numerical maximization techniques were used to maximize this non-linear log likelihood function based on the BFGS algorithm.

To estimate the model in equations (1) and (3), it is assumed that the vector of innovations is conditionally normal and a quasi-maximum likelihood method is applied. Bollerslev and

Wooldridge (1992) show that the standard errors calculated using this method are robust even when the normality assumption is violated.

3.2 Asymmetric Volatility Impulse Response Functions (AVIRF) with crisis

The Volatility Impulse-Response Function (VIRF), proposed by Lin (1997), is a useful methodology for obtaining information on the second moment interaction between related markets. The VIRF, AVIRF, and our proposed crisis version, measure the impact of an unexpected shock on the predicted volatility. This is:

$$R_{s,3} = \frac{\partial \text{vech} E[H_{t+s} | \psi_t]}{\partial dg(\varepsilon_t \varepsilon_t') } \quad (5)$$

where $R_{s,3}$ is a 3×2 matrix, $s = 1, 2, \dots$ is the lead indicator for the conditioning expectation operator, H_t is the 2×2 conditional covariance matrix, $\partial dg(\varepsilon_t \varepsilon_t') = (\varepsilon_{1,t}^2, \varepsilon_{2,t}^2)'$, ψ_t is the set of conditioning information. The *vech* operator transforms a symmetric $N \times N$ matrix into a vector by stacking each column of the matrix underneath the other, and considering only diagonal and lower diagonal elements of the matrix.

In volatility symmetric structures, it is not necessary to distinguish between positive and negative shocks, but with asymmetric structures the VIRF can change with the sign of the shock. The asymmetric VIRF (AVIRF) for the asymmetric BEKK model is introduced in Meneu and Torró (2003). Similarly, it would be interesting to distinguish between periods of relative stability and periods of financial distress. Therefore, in this article we introduce a version of the AVIRF which takes into account periods of stock market crisis. By applying (5) to (3), we obtain:

$$R_{s,3}^+ = \begin{cases} a & s = 1 \\ (a + b + 1/2g + \alpha w)R_{s-1,3}^+ & s > 1 \end{cases} \quad (6)$$

$$R_{s,3}^- = \begin{cases} a + g & s = 1 \\ (a + b + 1/2g + \alpha w)R_{s-1,3}^- & s > 1 \end{cases} \quad (7)$$

$$R_{s,3}^{+,c} = \begin{cases} a + w & s = 1 \\ (a + b + 1/2g + \alpha w)R_{s-1,3}^+ & s > 1 \end{cases} \quad (8)$$

$$R_{s,3}^{-,c} = \begin{cases} a + g + w & s = 1 \\ (a + b + 1/2g + \alpha w)R_{s-1,3}^- & s > 1 \end{cases} \quad (9)$$

where $R_{s,3}^+(R_{s,3}^-)$ represents the VIRF for positive (negative) initial shocks in periods of stability, $R_{s,3}^{+,c}(R_{s,3}^{-,c})$ represents the VIRF for positive (negative) initial shocks in periods of stock market crisis, a , b and g are 3x3 parameter matrices, α is the probability of occurrence of a crisis and w is a 3x3 parameter matrix that, in our case, equals s , m and l during the September 11, March 11 and July 7 terrorist attacks, respectively.⁹ Moreover, $a = D_N^+(A' \otimes A')D_N$, $b = D_N^+(B' \otimes B')D_N$, $g = D_N^+(G' \otimes G')D_N$ and $w = D_N^+(W' \otimes W')D_N$, where D_N is a duplication matrix, D_N^+ is its Moore-Penrose inverse and \otimes denotes the Kronecker product between matrices, that is:

$$D_N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D_N^+ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⁹ The specification in equations (6) to (9) for testing crises contagion can be seen as a particular case of the general framework proposed in Dungey et al. (2005). In our model, the "contagious transmission channel" (in words of Dungey et al. (2005)) is measured through the significance of w in the market receiving the crisis contagion.

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It is important to note that this impulse response function examines how fast asset prices can incorporate new information. This fact lets us test for the speed of adjustment, analyze the dependence of volatilities across the returns of the S&P500 and the EuroStoxx50, distinguish between negative and positive shocks and distinguish between crisis periods and non-crisis periods.

4 Empirical Results

4.1 Model estimation

Table 4 displays the estimated BEKK model of equation (3). To prevent this paper from becoming too long, the results of the estimated VAR(5) are not included, although they are available upon request. The low p-values obtained for most of the parameters show that the model fits the data well. Table 5 shows the standardized residuals analysis. It can be observed that the standardized residuals appear free from serial correlation and heteroskedasticity.

As mentioned above, the parameters of Table 4 cannot be interpreted individually. Instead, we have to focus on the non-linear functions that form the intercept terms and the coefficients of the lagged variance, covariance, and error terms. Table 6 displays the expected value and the standard errors of these non-linear functions.

Table 4. Estimation results

Multivariate GARCH model estimation

$$C = \begin{bmatrix} -0.001006 & 0.000017 \\ (0.00) & (0.70) \\ & -0.000511 \\ & (0.00) \end{bmatrix}$$

$$B = \begin{bmatrix} 0.950495 & 0.001525 \\ (0.00) & (0.07) \\ -0.008417 & 0.967856 \\ (0.00) & (0.00) \end{bmatrix}$$

$$A = \begin{bmatrix} -0.046510 & 0.118528 \\ (0.00) & (0.00) \\ 0.202174 & -0.098272 \\ (0.00) & (0.00) \end{bmatrix}$$

$$G = \begin{bmatrix} 0.295050 & 0.034440 \\ (0.00) & (0.00) \\ 0.105456 & 0.202836 \\ (0.00) & (0.00) \end{bmatrix}$$

$$S = \begin{bmatrix} 0.114195 & -0.018969 \\ (0.00) & (0.87) \\ -0.192744 & -0.152630 \\ (0.00) & (0.00) \end{bmatrix}$$

$$M = \begin{bmatrix} 0.042863 & 0.320945 \\ (0.43) & (0.00) \\ -0.105324 & 0.043739 \\ (0.23) & (0.69) \end{bmatrix}$$

$$L = \begin{bmatrix} -0.199620 & 1.307097 \\ (0.29) & (0.00) \\ 0.091490 & -6.347604 \\ (0.92) & (0.00) \end{bmatrix}$$

Note: This table shows the estimation of the model defined in equation (3). P-values appear in brackets. The necessary conditions for the stationarity of the process are satisfied.

Table 5. Summary statistics for the standardized residuals of the model

	$\varepsilon_{1,t} / \sqrt{h_{11,t}}$		$\varepsilon_{2,t} / \sqrt{h_{22,t}}$	
Q(12)	12.41548	[0.41291]	4.36900	[0.97582]
Q²(12)	11.23055	[0.50927]	13.43020	[0.33856]
ARCH(12)	5.903165	[0.92088]	7.484829	[0.82398]

Note: Q(12) and Q²(12) are Ljung-Box tests for twelfth order serial correlation in the standardized residuals and squared residuals. ARCH(12) is Engle's test for twelfth order ARCH, distributed as $\chi^2(12)$. The p-values of these tests are displayed as [.]

Table 6. Results of the linearized multivariate BEKK model

S&P500 conditional variance equation

$$h_{11,t} = 1,01 \times 10^{-6} + 0,9034 h_{11,t-1} - 0,0160 h_{12,t-1} + 7,0845 \times 10^{-5} h_{22,t-1} + 0,0021 \varepsilon_{1,t-1}^2 - 0,0188 \varepsilon_{1,t-1} \varepsilon_{2,t-1} + 0,0408 \varepsilon_{2,t-1}^2 + 0,0870 \eta_{1,t-1}^2 + 0,0617 \eta_{1,t-1} \eta_{2,t-1} + 0,0109 \eta_{2,t-1}^2 + 1,10 \times 10^{-7} \quad 0,0036 \quad 0,0019 \quad 1,7103 \quad 0,0005 \quad 0,0024 \quad 0,0031 \quad 0,0043 \quad 0,0069 \quad 0,0021$$

(9,1454) (244,72) (-8,3242) (4,1422) (3,9943) (-7,7722) (12,864) (19,968) (8,8912) (5,1711)

$$+ 0,01419 \delta_{1,t-1}^2 - 0,04592 \delta_{1,t-1} \delta_{2,t-1} + 0,0371 \delta_{2,t-1}^2 + 0,0018 \xi_{1,t-1}^2 - 0,0090 \xi_{1,t-1} \xi_{2,t-1} + 0,0112 \xi_{2,t-1}^2 + 0,0398 \vartheta_{1,t-1}^2 - 0,365 \vartheta_{1,t-1} \vartheta_{2,t-1} + 0,0083 \vartheta_{2,t-1}^2$$

0,0030 0,0028 0,0096 0,0047 0,0070 0,0188 0,0767 0,3712 0,1708

(4,6098) (-16,105) (3,8458) (0,3902) (-1,2803) (0,5957) (0,5190) (-0,0983) (0,0489)

EuroStoxx50 conditional variance equation

$$h_{22,t} = 2,61 \times 10^{-7} + 2,32 \times 10^{-6} h_{11,t-1} + 0,0029 h_{12,t-1} + 0,9387 h_{22,t-1} + 0,0140 \varepsilon_{1,t-1}^2 - 0,0232 \varepsilon_{1,t-1} \varepsilon_{2,t-1} + 0,0096 \varepsilon_{2,t-1}^2 + 0,0011 \eta_{1,t-1}^2 + 0,0139 \eta_{1,t-1} \eta_{2,t-1} + 0,0411 \eta_{2,t-1}^2 + 5,22 \times 10^{-7} \quad 2,61 \times 10^{-6} \quad 0,0016 \quad 0,0040 \quad 0,0006 \quad 0,0013 \quad 0,0010 \quad 0,0003 \quad 0,00245 \quad 0,0027$$

(0,5005) (0,8897) (1,7836) (232,43) (21,3179) (-17,7817) (9,3201) (3,3495) (5,7023) (14,7990)

$$+ 0,0003 \delta_{1,t-1}^2 - 0,0057 \delta_{1,t-1} \delta_{2,t-1} + 0,0232 \delta_{2,t-1}^2 + 0,1030 \xi_{1,t-1}^2 + 0,0280 \xi_{1,t-1} \xi_{2,t-1} + 0,0019 \xi_{2,t-1}^2 + 1,7085 \vartheta_{1,t-1}^2 - 16,5938 \vartheta_{1,t-1} \vartheta_{2,t-1} + 40,2920 \vartheta_{2,t-1}^2$$

0,0044 0,0362 0,0098 0,0209 0,0726 0,0097 0,1203 1,5037 4,8040

(0,0813) (-0,1599) (2,3750) (4,9072) (0,3865) (0,1960) (14,1995) (-11,0350) (8,3871)

Table 6. Results of the linearized multivariate BEKK model (continued)

S&P500- EuroStoxx50 conditional covariance equation									
$h_{12,t} = 1,71 \times 10^{-8} + 0,0014h_{11,t-1} + 0,9199h_{12,t-1} - 0,0081h_{22,t-1} - 0,0055\varepsilon_{1,t-1}^2 + 0,0046\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0,0199\varepsilon_{2,t-1}^2 + 0,0102\eta_{1,t-1}^2 + 0,0634\eta_{1,t-1}\eta_{2,t-1} + 0,0212\eta_{2,t-1}^2 -$									
$1,75 \times 10^{-8}$	0,0008	0,0037	0,0009	0,0007	0,0015	0,0011	0,0015	0,0023	0,0022
(0,9775)	(1,7832)	(248,7658)	(-8,3104)	(-7,9577)	(3,0283)	(-17,3108)	(6,6778)	(27,4310)	(9,6612)
$- 0,0023\delta_{1,t-1}^2 + 0,0218\delta_{1,t-1}\delta_{2,t-1} - 0,0294\delta_{2,t-1}^2 + 0,0138\xi_{1,t-1}^2 - 0,0321\xi_{1,t-1}\xi_{2,t-1} - 0,0046\xi_{2,t-1}^2 - 0,2609\vartheta_{1,t-1}^2 + 1,3867\vartheta_{1,t-1}\vartheta_{2,t-1} - 0,5807\vartheta_{2,t-1}^2$									
0,0137	0,0232	0,0079	0,0166	0,0308	0,0128	0,2548	1,7489	5,9132	
(-0,1652)	(0,9428)	(-3,7294)	(0,8297)	(-1,0429)	(-0,3606)	(-1,0240)	(0,7929)	(-0,0982)	

Note: h_{11} and h_{22} denote the conditional variance for the S&P500 and EuroStoxx50 return series, respectively. Below the estimated coefficients are the standard errors, with the corresponding t-values given in parentheses.

The expected value is obtained by taking expectations to the non-linear functions, therefore involving the estimated variance-covariance matrix of the parameters. To calculate the standard errors, the function must be linearised using first order Taylor series expansion. This is sometimes called the 'delta method'. When a variable Y is a function of a variable X, i.e., $Y = F(X)$, the delta method enables us to obtain approximate formulation of the variance of Y if: (1) Y is differentiable with respect to X and (2) the variance of X is known. Therefore:

$$V(Y) \approx (\Delta Y)^2 \approx \left(\frac{\partial Y}{\partial X}\right)^2 (\Delta X)^2 \approx \left(\frac{\partial Y}{\partial X}\right)^2 V(X)$$

When a variable Y is a function of variables X and Z in the form of $Y = F(X, Z)$, we can obtain approximate formulation of the variance of Y if: (1) Y is differentiable with respect to X and Z and (2) the variance of X and Z and the covariance between X and Z are known. This is:

$$V(Y) \approx \left(\frac{\partial Y}{\partial X}\right)^2 V(X) + \left(\frac{\partial Y}{\partial Z}\right)^2 V(Z) + 2\left(\frac{\partial Y}{\partial X}\right)\left(\frac{\partial Y}{\partial Z}\right)Cov(X, Z)$$

Once the variances are calculated it is straightforward to calculate the standard errors.

The S&P500 volatility is directly affected by its own volatility ($h_{1,1}$) and by the EuroStoxx50 volatility ($h_{2,2}$). Our findings suggest that the S&P500 volatility is affected by its own shocks (ε_1^2) and the EuroStoxx50 shocks (ε_2^2). Finally, the coefficient for its own asymmetric term (η_1^2) and the EuroStoxx50 asymmetric term are significant (η_2^2), indicating that negative shocks affect market volatility more than positive shocks.

The behaviour of the EuroStoxx50 volatility does not differ much from that of the S&P500. The EuroStoxx50 volatility is affected by its own volatility ($h_{2,2}$), but not by the S&P500 volatility.¹⁰ Interestingly, the EuroStoxx50 volatility is affected by the S&P500 shocks (ε_1^2) and its own shocks (ε_2^2). Finally, the coefficient for its own asymmetric term (η_1^2) and the EuroStoxx50 asymmetric term are significant (η_2^2), indicating that negative shocks affect market volatility more than positive shocks.

The covariance between the S&P500 and the EuroStoxx50 is affected by its own past values ($h_{1,2}$), the EuroStoxx50 volatility ($h_{2,2}$), the S&P500 shocks (ε_1^2) and the EuroStoxx50 shocks (ε_2^2). Moreover, the coefficients η_1^2 and η_2^2 are significant, indicating that negative shocks affect market covariance more than positive shocks.

Regarding dummies, from the analysis of the coefficients significance, the most appealing results are: (1) the September 11 terrorist attack had an influence over volatility in both the US and Eurozone markets, although in the case of the Eurozone, the effect was indirectly transmitted through its own shocks. (2) Both the March 11 and July 7 terrorist attacks did not affect the S&P500 volatility. (3) The July 7 terrorist attack in London had an effect on volatility in the

¹⁰ This could be due to the fact that prices are recorded at 15:00 GMT, when European markets are about to close and the US market has just started trading.

Eurozone. However, the March 11 terrorist attack only affected volatility in the Eurozone indirectly through shocks coming from the S&P500.

In general, there is bidirectional volatility transmission between the US and the Eurozone stock markets. However, the terrorist attack in New York on September 11 affected volatility in the Eurozone stock markets, but the terrorist attacks in Madrid and London in March 11 and July 7 respectively did not affect volatility in the US market.

4.2 Asymmetric Volatility Impulse Response Functions (AVIRF) with crisis

Figures 1 to 5 present the AVIRFs with crisis, computed following Lin (1997) and Meneu and Torró (2003), as explained in section 3.2. Results add evidence in favour of bidirectional volatility transmission between the US and Eurozone stock markets and show the different impacts of these terrorist attacks on both markets. These graphical representations also allow us to test for the speed of adjustment, analyze the dependence of volatilities across the returns of the S&P500 and the EuroStoxx50, distinguish between negative and positive shocks, and distinguish between crisis periods and non-crisis periods.¹¹

Figure 1 represents the AVIRF when unexpected shocks are positive and there is a period of financial stability as opposed to stock market crisis periods caused by terrorist attacks. The graphical analysis shows that there are bidirectional volatility spillovers between the S&P500 and the EuroStoxx50 (about 4% and 1.5% of the shock, respectively, Figures 1B and 1C). Positive shocks in the EuroStoxx50 have a relatively small effect on its own volatility (Figure

¹¹ The standard deviation of the responses depends on the estimated coefficients and their covariance matrix (see equation (8) in Lin (1997)). Exploding confidence bands in Figures 5A and 5C are due to the high values of estimated coefficients in matrix L (see Table 4), even though GARCH coefficients satisfy stationarity conditions. This instability can be avoided in several ways, for instance, by enlarging the number of days that this dummy takes values different from zero. Nevertheless, when this is done, no coefficient in L remains statistically significant.

1D), whereas past positive shocks in the S&P500 have no effect on current volatility (Figure 1A).

If unexpected shocks are negative and there is a period of financial stability, Figure 2 shows that there are also bidirectional volatility spillovers between the S&P500 and the EuroStoxx50 (Figures 2B and 2C). Negative shocks in the S&P500 have an important effect on its own volatility (Figure 2A). Negative shocks in the EuroStoxx50 also have an important effect on its own volatility (Figure 2D), though they are less important than in the case of the S&P500. It is interesting to note that own positive shocks do not have any effect on S&P500 volatility, whereas own negative shocks have a very significant effect. In all cases, there is evidence of asymmetry: negative shocks have a greater effect on volatility than positive shocks. The only exception is the effect of shocks from the S&P500 on the EuroStoxx50, where both kinds of shock have a similar and relatively small impact on volatility.

One of the most appealing contributions of the new version of the AVIRF introduced in this paper is that it enables differentiation between periods of relative financial stability and periods of stock market crisis caused, in this case, by terrorist attacks. Figure 3 represents the AVIRF to negative unexpected shocks during the crisis period produced by the September 11 terrorist attack. Similarly, Figures 4 and 5 represent the AVIRF to negative unexpected shocks during the March 11 and July 7 crisis periods, respectively. To interpret these graphs, it is important to compare the figures with those obtained in Figure 2, AVIRF to negative unexpected shocks in a no-crisis period.

In general, the most appealing results are: (1) conditional variances are more sensitive to negative than positive shocks; (2) the September 11 terrorist attack (Figure 3) had an influence over volatility of both the US and Eurozone markets, because all figures increased their initial

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6 response to a shock when compared to Figure 2. In the case of the Eurozone, the effect was
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8 indirectly transmitted through its own shocks (Figure 3D). (3) Both the March 11 and July 7
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10 terrorist attacks did not affect the S&P500 volatility (Figures 4A, 4B, 5A and 5B are either non-
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12 significant or they do not change when compared to Figure 2). (4) The March 11 and July 7
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14 terrorist attacks had an effect on volatility in the Eurozone (Figures 4C, 4D, 5C and 5D).
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16 However, the March 11 terrorist attack (Figure 4) only affected volatility in the Eurozone
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18 indirectly through shocks coming from the S&P500 (Figure 4C), as Figure 4D does not change
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20 when compared to Figure 2D.
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25 Therefore, these results add evidence favouring the hypothesis of bidirectional variance
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27 causality between the S&P500 and the EuroStoxx50, but also favouring the hypothesis of
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29 differing reactions to the terrorist attack from each stock market.
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Figure 1A. A positive shock in the S&P500

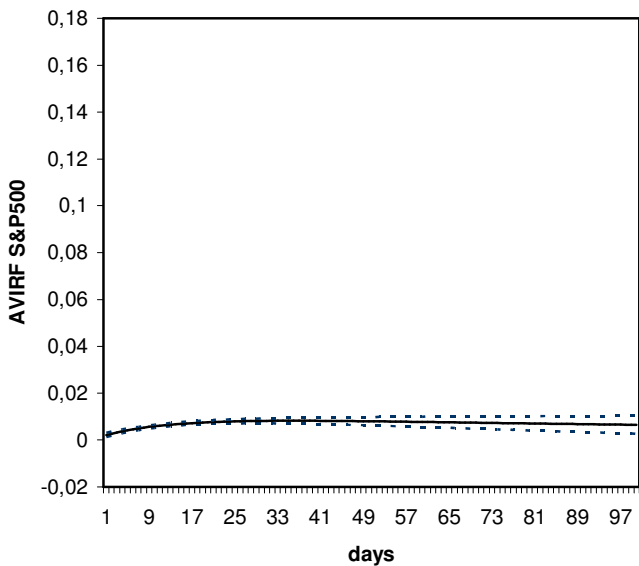


Figure 1B. A positive shock in the EuroStoxx50

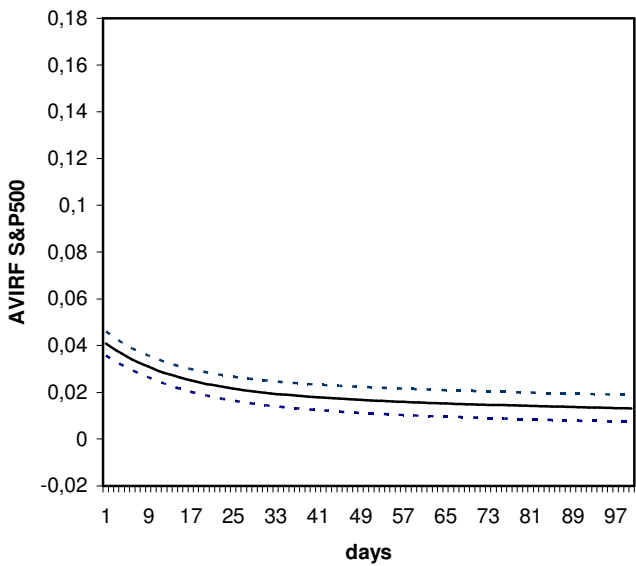


Figure 1C. A positive shock in the S&P500

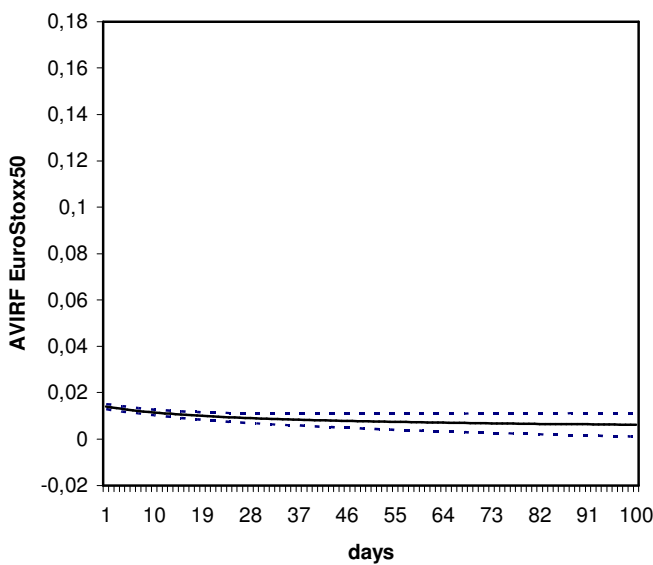


Figure 1D. A positive shock in the EuroStoxx50

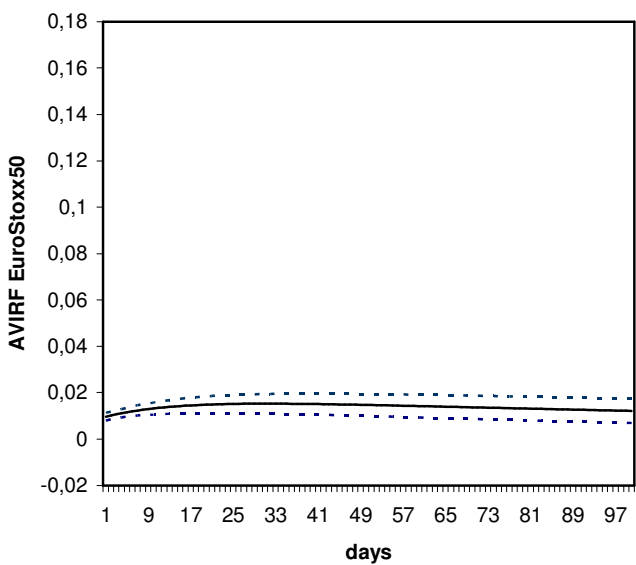


Figure 1. AVIRF to positive unexpected shocks from the VAR-Asymmetric BEKK
No Crisis Period
(Dashed lines display the 90% confidence interval)

Figure 2A. A negative shock in the S&P500

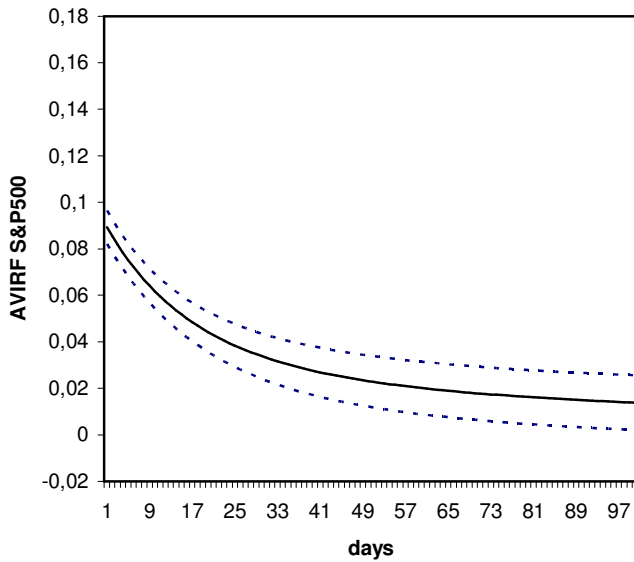


Figure 2B. A negative shock in the EuroStoxx50

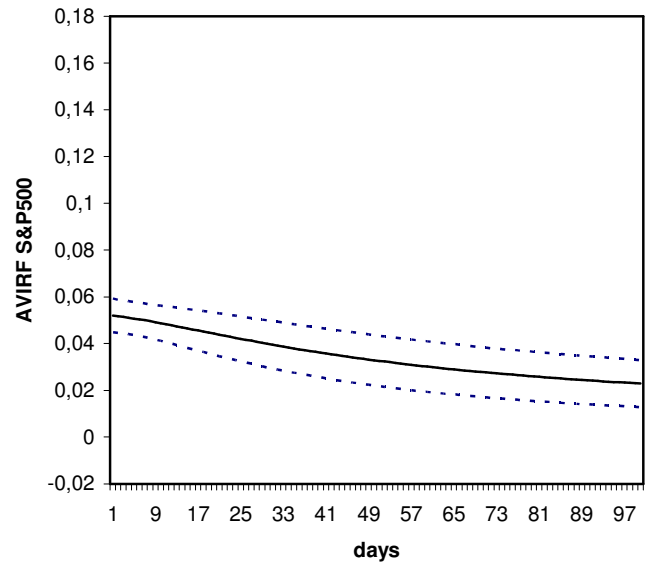


Figure 2C. A negative shock in the S&P500

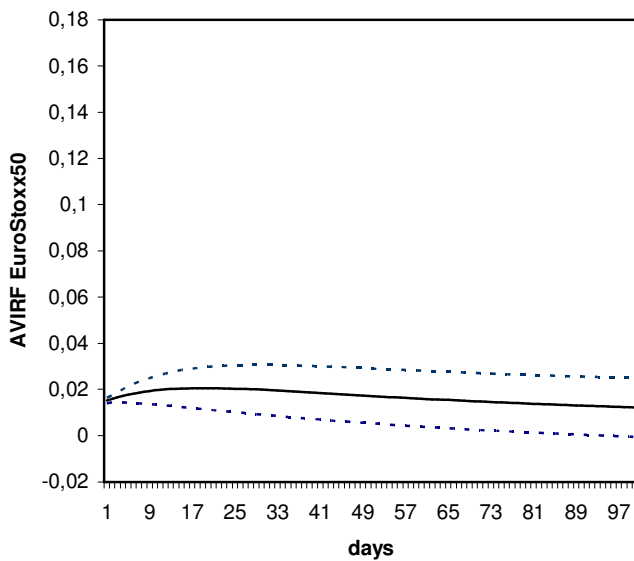


Figure 2D. A negative shock in the EuroStoxx50

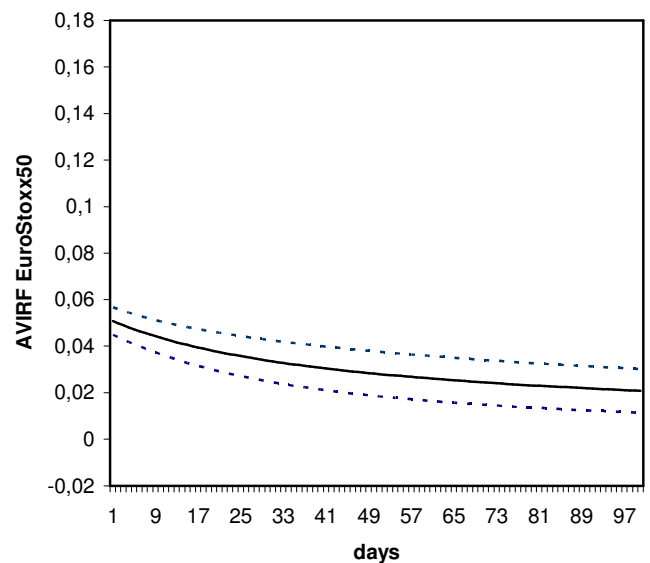


Figure 2. AVIRF to negative unexpected shocks from the VAR-Asymmetric BEKK
No Crisis Period
 (Dashed lines display the 90% confidence interval)

Figure 3A. A negative shock in the S&P500

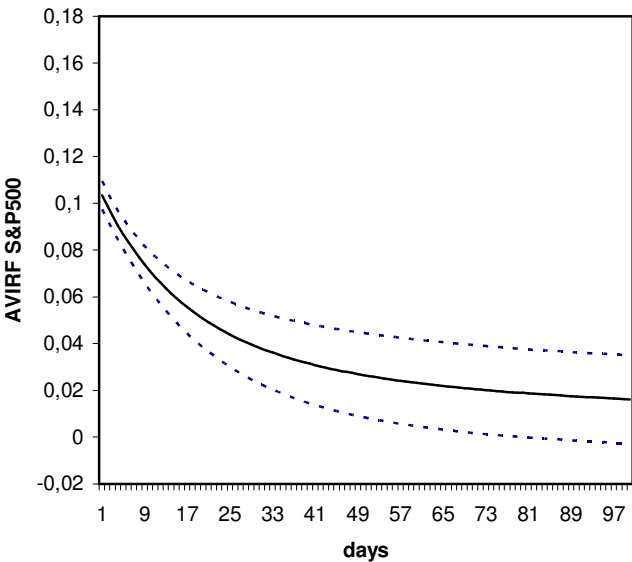


Figure 3B. A negative shock in the EuroStoxx50

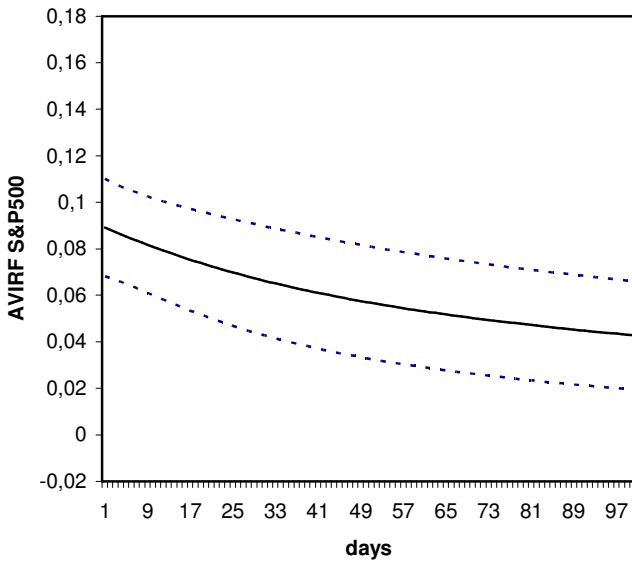


Figure 3C. A negative shock in the S&P500

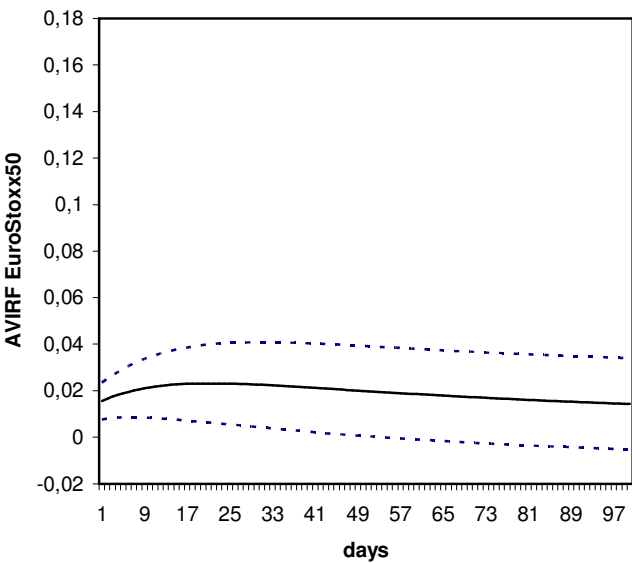


Figure 3D. A negative shock in the EuroStoxx50

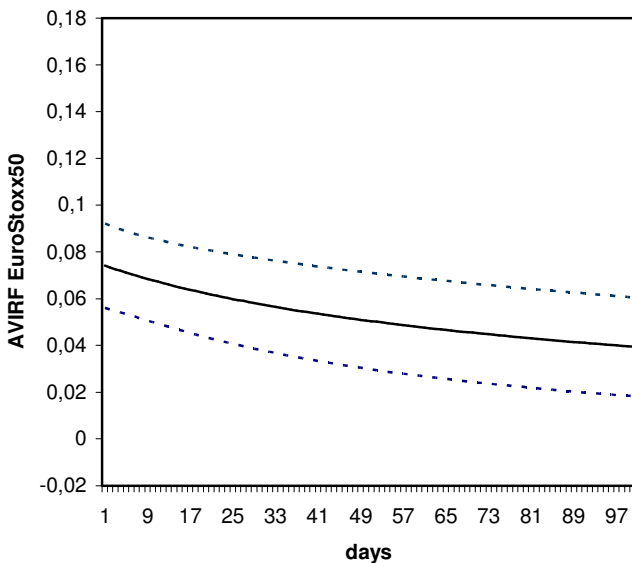


Figure 3. AVIRF to negative unexpected shocks from the VAR-Asymmetric BEKK
Crisis Period (September 11)
(Dashed lines display the 90% confidence interval)

Figure 4A. A negative shock in the S&P500

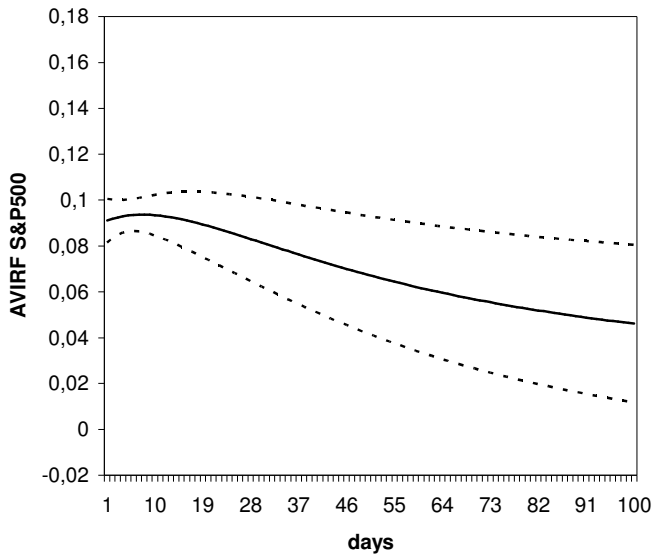


Figure 4B. A negative shock in the EuroStoxx50

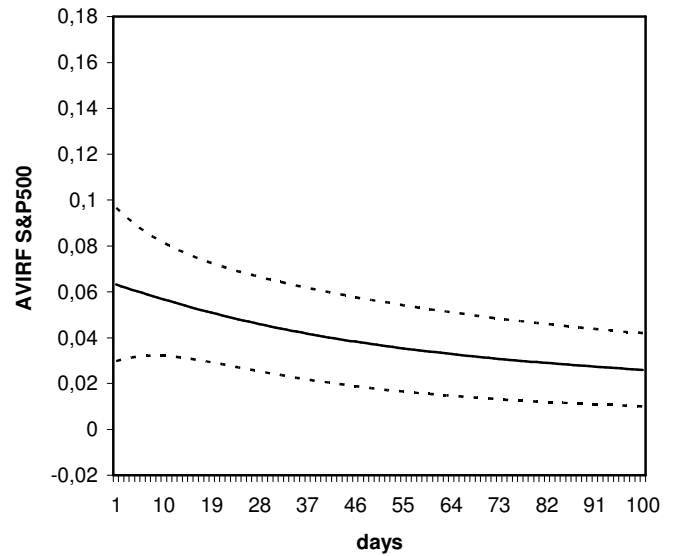


Figure 4C. A negative shock in the S&P500

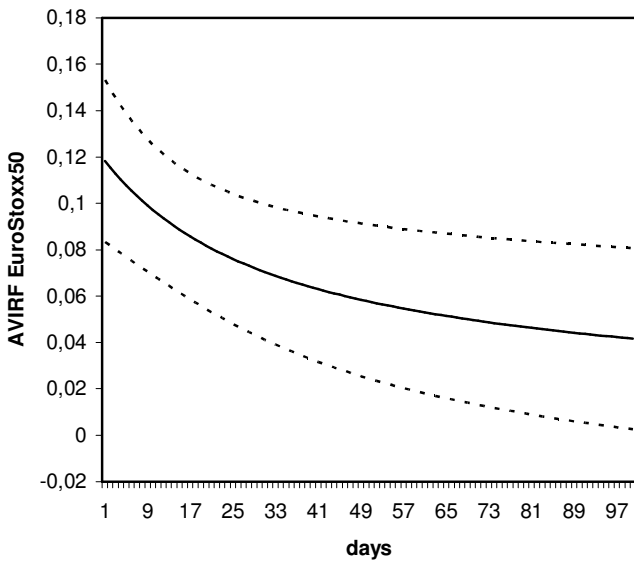
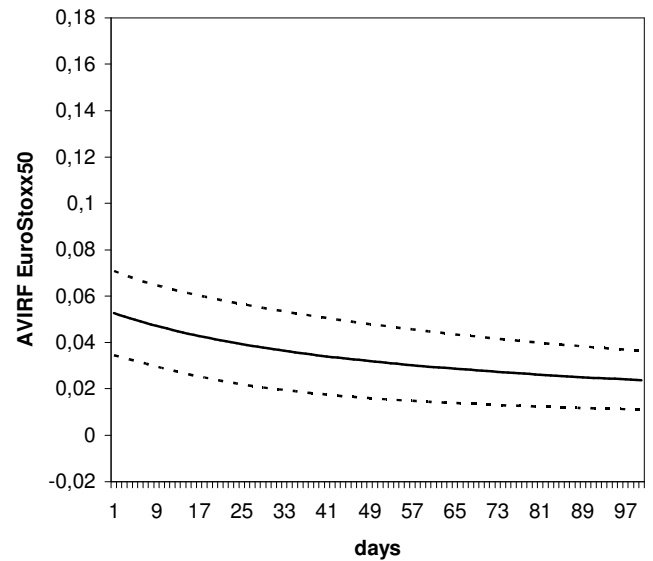


Figure 4D. A negative shock in the EuroStoxx50



**Figure 4. AVIRF to negative unexpected shocks from the VAR-Asymmetric BEKK
Crisis Period (March 11)**
(Dashed lines display the 90% confidence interval)

Figure 5A. A negative shock in the S&P500

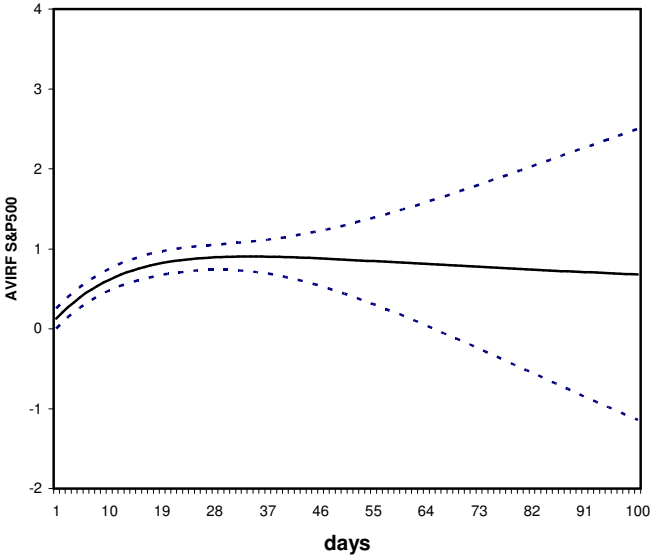


Figure 5B. A negative shock in the EuroStoxx50

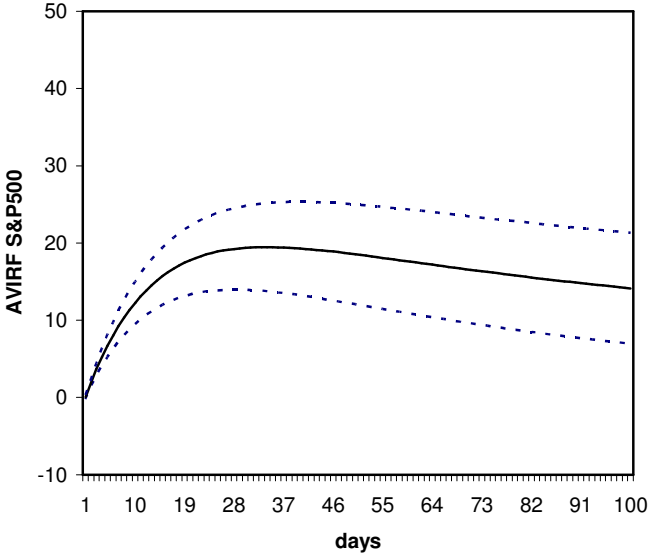


Figure 5C. A negative shock in the S&P500

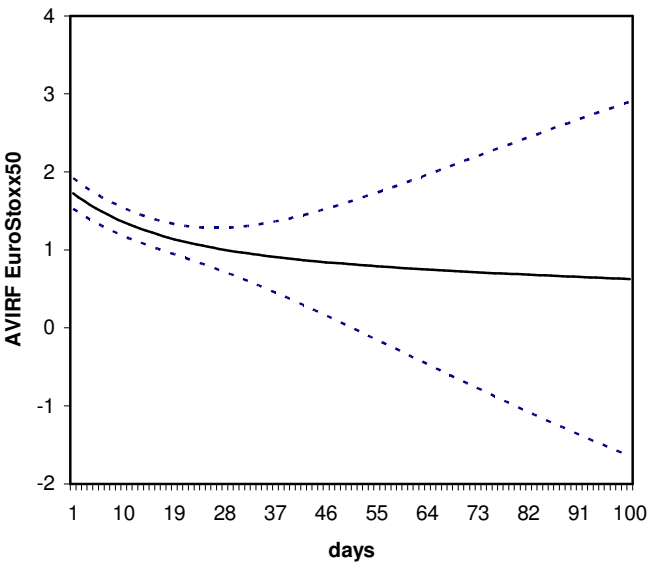
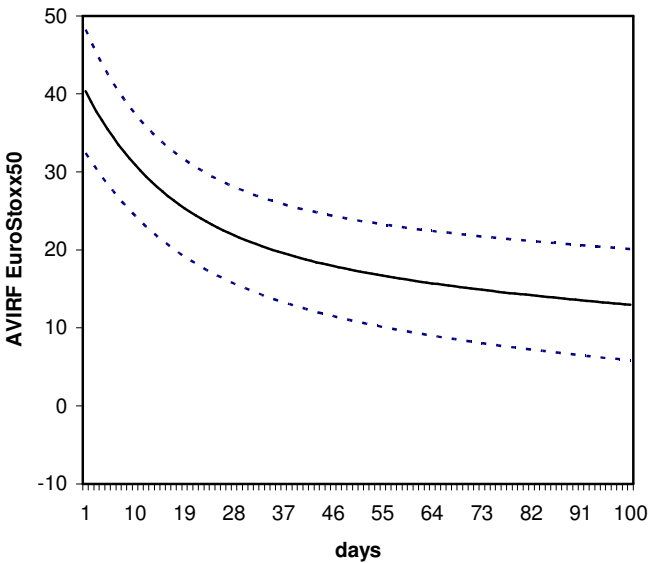


Figure 5D. A negative shock in the EuroStoxx50



**Figure 5. AVIRF to negative unexpected shocks from the VAR-Asymmetric BEKK
Crisis Period (July 7)**
(Dashed lines display the 90% confidence interval)

5 Conclusions

The main objective of this study is to analyze how volatility transmission patterns are affected by stock market crises. To do this, we use a multivariate GARCH model and take into account both the asymmetric volatility phenomenon, and the non-synchronous trading problem. In our empirical application, we focus on stock market crises as a result of terrorist attacks and analyze international volatility transmission between the US and Eurozone financial markets.

In particular, an asymmetric VAR-BEKK model is estimated with daily stock market prices recorded at 15:00 GMT for the US (S&P500 index) and Eurozone (EuroStoxx50 index).

We also introduce a complementary analysis, the Asymmetric Volatility Impulse Response Functions (AVIRF) with crisis, which distinguishes both a) effects coming from a positive shock from those coming from a negative shock, and b) effects coming from periods of stability from those coming from periods of crisis.

The results confirm that there are asymmetric volatility effects on both markets and that volatility transmission between the US and the Eurozone is bidirectional. The terrorist attack in New York in September 11 affected volatility in the Eurozone stock markets, but the terrorist attacks in Madrid and London in March 11 and July 7, respectively, did not affect volatility in the US market.

Based on Johnston and Nedelescu (2006), there are several possible explanations for the differences in stock market reactions to the three terrorist attacks considered. Firstly, the September 11 terrorist attack had a direct impact on several financial markets, such as the aeronautical, tourism, banking, and insurance sectors. These sectors were not so badly affected in the case of the other terrorist attacks considered. Secondly, while the attacks in New York were perceived as a global shock, the attacks on Madrid and London were perceived as mostly having

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a local and regional effect, respectively. Finally, while the events of September 11 occurred in the midst of a global economic downturn, the terrorist attacks in Madrid and London occurred at a time when the world economy was growing strongly.

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References

- Allen, F. and D. Gale. (2000) "Financial Contagion." *Journal of Political Economy*, Vol. 108, pp. 1-33.
- Baba, Y.; R.F. Engle, D.F. Kraft, and K.F. Kroner. (1989) "Multivariate Simultaneous Generalized ARCH." Mimeo, University of California at San Diego.
- Baele, L. (2005) "Volatility Spillover Effects in European Equity Markets." *Journal of Financial and Quantitative Analysis*, Vol. 40, No. 2, pp. 373-402.
- Bera, A.K. and S. Kim. (2002) "Testing Constancy of Correlation and other Specifications of the BGARCH Model with an Application to International Equity Returns." *Journal of Empirical Finance*, Vol. 9, No. 2, pp. 171-195.
- Blunk, S.S.; D.E. Clark and J.M. McGibany. (2006) "Evaluating the Long-Run Impacts of the 9/11 Terrorist Attacks on US Domestic Airline Travel." *Applied Economics*, Vol. 38, No. 4, pp. 363-370.
- Bollerslev, T. and J.M. Wooldridge. (1992) "Quasi-maximum Likelihood Estimation and Inference in Dynamic Models with Time-varying Covariances." *Econometric Reviews*, Vol. 11, No. 2, pp. 143-172.
- Brooks, C. and O.T. Henry. (2000) "Linear and Non-linear Transmission of Equity Return Volatility: Evidence from the US, Japan and Australia." *Economic Modelling*, Vol. 17, No. 4, pp. 497-513.
- Caporale, G. M., A. Cipollini and N. Spagnolo. (2004) "Testing for contagion: a conditional correlation analysis." *Journal of Empirical Finance*, Vol. 12, pp. 476-489.
- Chen, A.H and T.F. Siems. (2004) "The Effects of Terrorism on Global Capital Markets." *European Journal of Political Economy*, Vol. 20, No. 2, pp. 349-366.

- Choudhry, T. (2005) "September 11 and Time-Varying Beta of United States Companies." *Applied Financial Economics*, Vol. 15, No. 17, pp. 1227-1242.
- Darbar, S.M. and P. Deb. (1997) "Co-movements in International Equity Markets." *Journal of Financial Research*, Vol. 20, No.3, pp. 305-322.
- Dickey, D. A. and W. A. Fuller. (1981) "Likelihood ratios statistics for autorregressive time series with a unit root." *Econometrica*, Vol. 49, pp. 1057-72.
- Dungey, M., M. Fry, B. Gonzalez-Hermosillo and V. L. Martin. (2005) "Empirical modelling of contagion: a review of methodologies." *Quantitative Finance*, Vol. 5, No. 1, pp. 9-24.
- Dungey, M. and V. L. Martin. (2007) "Unravelling financial market linkages during crises." *Journal of Applied Econometrics*, Vol. 22, pp. 89-119.
- Engle, R.F. and K.F. Kroner. (1995) "Multivariate Simultaneous Generalized ARCH." *Econometric Theory*, Vol. 11, No. 1, pp. 122-150.
- Glaser, M. and M. Weber. (2006) "September 11 and stock returns expectations of individual investors." *Review of Finance*, Fothcoming.
- Glosten, L.R., R. Jagannathan, and D.E. Runkle. (1993) "On the Relationship between the Expected Value and the Volatility of the Nominal Excess Return on Stocks." *Journal of Finance*, Vol. 48, No. 5, pp. 1779-1801.
- Hamao, Y., R.W. Masulis, and V. Ng. (1990) "Correlations in Price Changes and Volatility across International Stock Markets." *Review of Financial Studies*, Vol. 3, No. 2, pp. 281-307.
- Hon, M. T.; J. Strauss and S. Yong. (2004) "Contagion in financial markets after September 11: myth or reality?" *Journal of Financial Research*, Vol. 27, No. 1, pp. 95-114.
- Ito, H. and D. Lee. (2005) "Assesing the impact of September 11 terrorist attacks on U.S. airline demand." *Journal of Economics and Business*, Vol. 57, pp. 75-95.

- Johnston, B. and O. Nedelescu. (2006) "The impact of terrorism on financial markets." *Journal of Financial Crime*, Vol. 13, No. 6, pp. 7-25.
- Karolyi, G.A. (1995) "A Multivariate GARCH Model of International Transmissions of Stock Returns and Volatility: the Case of the United States and Canada." *Journal of Business and Economic Statistics*, Vol. 13, No. 1, pp. 11-25.
- Karolyi, G.A. and R.M Stulz. (1996) "Why Do Markets Move Together? An Investigation of US-Japan Stock Return Comovements." *Journal of Finance*, Vol. 51, No. 3, pp. 951-986.
- Kearney, C. and A.J. Patton. (2000) "Multivariate GARCH Modeling of Exchange Rate Volatility Transmission in the European Monetary System." *Financial Review*, Vol. 35, No.1, pp. 29-48.
- Kleimeier, S; T. Lehnert and W.F.C. Verschoor. (2008) "Measuring financial contagion using time-aligned data: the importance of the speed of transmission of shocks." *Oxford Bulletin of Economics and Statistics*, Forthcoming.
- Koutmos, G. and G. Booth. (1995) "Asymmetric Volatility Transmission in International Stock Markets." *Journal of International Money and Finance*, Vol. 14, No. 6, pp. 747-762.
- Kroner, K.F. and V.K. Ng. (1998) "Modeling Asymmetric Comovements of Asset Returns." *Review of Financial Studies*, Vol. 11, No. 4, pp. 817-844.
- Lin, W.L. (1997) "Impulse Response Function for Conditional Volatility in GARCH Models." *Journal of Business and Economic Statistics*, Vol. 15, No. 1, pp. 15-25.
- Longin, F. and B. Solnik. (1995) "Is the correlation in international equity returns constant: 1960-1990?" *Journal of International Money and Finance*, Vol. 14, No. 1, pp. 3-26.
- Longin, F. and B. Solnik. (2001) "Extreme Correlation of International Equity Markets." *Journal of Finance*, Vol. 56, No. 2, pp. 649-676.

- MacKinnon, J.G. (1991) "Critical values for cointegration Tests." Chapter 13 in *Long-run Economic Relationships: Readings in Cointegration*, edited by R.F. Engle and C. W. J. Granger, Oxford University Press.
- MacKinnon, J.G.; A.A. Haug, and L. Michelis. (1999) "Numerical Distribution Functions of Likelihood Ratio Tests for Cointegration." *Journal of Applied Econometrics*, Vol. 14, No. 5, pp. 563-577.
- Martens, M. and S.H. Poon. (2001) "Returns Synchronization and Daily Correlation Dynamics between International Stock Markets." *Journal of Banking and Finance*, Vol. 25, No. 10, pp. 1805-1827.
- Meneu, V. and H. Torró. (2003) "Asymmetric Covariance in Spot-Futures Markets." *Journal of Futures Markets*, Vol. 23, No. 11, pp. 1019-1043.
- Mun, C.K. (2005) "Contagion and Impulse Response of International Stock Markets around the 9-11 Terrorist Attacks." *Global Finance Journal*, Vol. 16, No. 1, pp. 48-68.
- Pericoli, M., and M. Sbracia. (2003) "A Primer on financial contagion." *Journal of Economic Surveys*, Vol. 17, No. 4, pp. 571-608.
- Philips, P. C. B. and P. Perron. (1988) "Testing for a unit root in time series regression." *Biometrika*, Vol. 75, pp. 335-346.
- Poteshman, A. M (2006) "Unusual option market activity and the terrorist attacks of September 11, 2001." *Journal of Business*, Vol. 79, No. 4, pp. 1703-1726.
- Ramchand, L. and R. Susmel. (1998) "Volatility and Cross Correlation across Major Stock Markets." *Journal of Empirical Finance*, Vol. 5, No. 4, pp. 397-416.
- Wongswan, J. (2006) "Transmission of information across international equity markets." *Review of Financial Studies*, Vol. 19, No. 4, pp.1157-1189.